15.

a.

Nearby Electromagnetic Observation belongs to NP-complete class because given a set of k locations, there is no polynomial time algorithm to determine the locations where frequency is unblocked. However, this solution can be verified in polynomial time.

b.

I chose vertex cover problem to be X; it is a NP-Complete. I prove that Nearby Electromagnetic Observation, call it Y, is NP-Complete by reduction; or in other words,

A problem 𝑌 ∈ 𝑁𝑃 with the property that for every 𝑋 ∈ 𝑁𝑃, 𝑋 ≤ p𝑌.

𝑉𝑒𝑟𝑡𝑒𝑥 𝐶𝑜𝑣𝑒𝑟 ≤ p 𝑁𝑒𝑎𝑟𝑏𝑦 𝐸𝑙𝑒𝑐𝑡𝑟𝑜𝑚𝑎𝑔𝑛𝑒𝑡𝑖𝑐 𝑂𝑏𝑠𝑒𝑟𝑣𝑎𝑡𝑖𝑜𝑛.

I define a Graph 𝐺 = (𝑉, 𝐸) and an integer k. We want to find a subset of vertices 𝑆 ⊆ 𝑉 such

that |𝑆| ≥ 𝑘, and for each edge, at most one of its endpoints is in S. Note that the subsets of S are vertex covers.

Let each location 𝑙𝑖 correspond to each node and 𝑓ℎ correspond to each edge.

For an edge at (𝑙𝑥 , 𝑙𝑦 ), there is an interference source that blocks 𝑓ℎ at all but locations 𝑙𝑥 and 𝑙𝑦.

If there is a vertex cover of at least size k, then each frequency does not have any interference in at least one of the locations. This is equivalent to the vertex cover that for the set S of nodes, each edge has at most one of its endpoints in S. Therefore, a vertex cover S exists in graph G.

Therefore, we can conclude that

𝑉𝑒𝑟𝑡𝑒𝑥 𝐶𝑜𝑣𝑒𝑟 ≤ p𝑁𝑒𝑎𝑟𝑏𝑦 𝐸𝑙𝑒𝑐𝑡𝑟𝑜𝑚𝑎𝑔𝑛𝑒𝑡𝑖𝑐 𝑂𝑏𝑠𝑒𝑟𝑣𝑎𝑡𝑖𝑜𝑛.

This proves that the Nearby Electromagnetic Observation is NP-complete.

22.

Based on the given information:

If 𝑉 = ∅ , then G contains an independent set because it has no edges.

If 𝑉 ≠ ∅ and 𝑘 > 1, I Add an extra node 𝑣′ to G and add one edge between 𝑣′ and each node in G. Call the new graph 𝐺′.

Use the given black-box algorithm 𝐴 to see if 𝐺′ has an independent set of at least size k.

If G has an independent set of size at least k, then 𝐺′ also has an independent set of at least k.

G and 𝐺′ have the same independent set of size at least k because 𝑣′ can be considered another

independent set in 𝐺′ since 𝑣′ is connected to each node only once.

G only has an independent set of size at least k if 𝐺′ does.

Therefore, the Independent Set Problem is solved in polynomial time since A is called once and runs in polynomial time

36.

We can show that a Hamilton Cycle reduces to the Daily Special Scheduling.

In other words, 𝐻𝑎𝑚𝑖𝑙𝑡𝑜𝑛 𝐶𝑦𝑐𝑙𝑒 ≤ p𝐷𝑎𝑖𝑙𝑦 𝑆𝑝𝑒𝑐𝑖𝑎𝑙 𝑆𝑐ℎ𝑒𝑑𝑢𝑙𝑖𝑛𝑔.

Consider the graph 𝐺 = (𝑉, 𝐸). Let each node correspond to a special; let each edge correspond

to an ingredient 𝐼𝑛. Edges 𝐼𝑛 are incident on a node that requires those ingredients.

The goal is to find a Hamilton Cycle in G that uses up the total money at most x.

Consider i edges and j nodes for the graph G. The cost is twice the ingredients minus the nodes

because each ingredient can be reused for another node. Thus, the cost of a Hamilton Cycle is

2𝑖 − 𝑗 + 1. For every pair of special nodes, there is an edge between them. As a result, there is a

simple cycle that can transverse once through without repeating any nodes. Thus, the Hamilton

Cycle reduces to the Daily Special Scheduling.

This makes the Daily Special Scheduling NP-complete